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Discussion

Vibration behavior of ACLD treated beams under thermal environment

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1. Introduction

Sandwich structures are heavily used as sub-components in the construction of airplane, missile and spacecraft structures. Sandwich structures with viscoelastic cores are particularly useful in vibration damping over a wide range of frequencies. This is known as passive constrained layer damping (PCLD) treatment. Though viscoelastic materials are highly reliable in suppressing vibration they add additional weight to the structure, which puts a limit to their usage beyond a certain limit. Piezoelectric actuators on the other hand are efficient in suppressing vibration without adding much of additional weight to the structure. This is known as active layer damping (ALD) treatment. However, these materials are inefficient at high-frequency range and are not highly reliable. Hybrid damping treatment, popularly known as active constrained layer damping (ACLD) judiciously combines the advantages of the two, by having a viscoelastic core sandwiched between the base beam and active piezoelectric constrained layer. Khatua and Cheung [1] presented a finite element formulation for bending and vibration of multilayer sandwich beams and plates, with constrained cores. Rao and Nakra [2] carried out analysis of vibration of unsymmetrical sandwich beams and plates with viscoelastic cores. Rao [3] derived the complete set of equations of motion and boundary conditions governing the vibration of sandwich beams using energy approach. Park and Baz [4] studied the dynamics of plates treated with ACLD and compared the finite element formulations obtained using two theories namely classical laminate theory and layerwise laminate theory. Balamurugan and Narayanan [5] presented a finite element formulation and active vibration control of beams with smart constrained layer damping. In their work the frequency dependent characteristics of viscoelastic materials have been accounted by the well-known GHM method. Hau and Fung [6] studied the effect of ACLD treatment configuration on damping performance of a flexible beam. They have used equivalent single layer theory for the combined base and sensor layers and individual layer theory for the sandwich beam system. They have incorporated frequency dependent characteristics of viscoelastic layer (VEL) by using GHM method. In general viscoelastic material properties are dependent both on the frequency and temperature. Many researchers have focused their attention towards incorporating the frequency dependent characteristics in modeling. But there exists very few papers incorporating the temperature dependent material properties of VEL. Ganesan and Pradeep [7] have

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considered the buckling and vibration behavior of viscoelastic sandwich beams under thermal environment considering the temperature dependence of shear modulus of the VEL core. Trindade et al. [8] have given a finite element formulation for frequency–temperature dependent hybrid damping. However, their study ignores the initial stress effects because of the thermal environment. To the author's knowledge, there is no work reported in the literature on hybrid damping under thermal environment considering thermally induced pre-stresses and temperature dependent shear modulus of the core. The present work deals with a clamped–clamped ACLD beam subjected to a tip temperature. The loss factor variation and frequency variation with respect to temperature considering the temperature dependent core shear modulus and temperature dependent material loss factor of the core are reported. Two different core VEL materials are considered. A parametric study with thickness of the core, core material and control gain has been conducted. The relative comparison between active and passive damping has been attempted.

2. Finite element formulation

To evaluate the damping and vibration behaviors of the beam under thermal environment, thermally induced pre-stresses in the beam have to be calculated. The present formulation is a de-coupled thermo-mechanical formulation. The temperature field in the beam is calculated by using 4 noded rectangular elements. The sensor is rigidly bonded to the base beam and in between the sensor and constraining layer is the VEL as shown in Fig. 1.

The following are the assumptions made in constructing the finite element model:

1. The sensor layer is perfectly bonded to the base beam.
2. The combined base beam and sensor layer will behave like an equivalent single layer.
3. The transverse shears in the stiff layers are neglected.
4. The core material is viscoelastic with temperature dependent complex shear modulus (for details see Ref. [10]).

Fig. 1 shows a beam treated with ACLD along with the degrees of freedom used for modeling the beam. The expressions for stiffness matrix, mass matrix, thermal load vector and geometric stiffness matrix are not presented for brevity. These are well-known expressions given by Kautua et al. and Ganesan et al. [1,7].

The equation of motion of the sandwich beam under thermal environment is given by

$$[M]\{\ddot{\delta}_G\} + [C]\{\dot{\delta}_G\} + [[K] + [K_G]]\{\delta_G\} = 0, \tag{1}$$

where $[M]$, $[C]$, $[K]$, $[K_G]$, $\{\delta_G\}$ are the elemental mass matrix, piezoelectric damping matrix, complex stiffness matrix resulting due to viscoelastic core, geometric stiffness matrix, and array of global displacements, respectively. The averaged sensor voltage over an element is given by [9]

$$\bar{\phi}_s = \frac{-t_s b}{\varepsilon_{33} A_s}\left(\int_x \left\{ e_{31} \quad \frac{t_b + t_s}{2} e_{31} \quad 0 \quad 0 \quad 0 \right\} [B]\{\delta\}\,\mathrm{d}x + \int_x \{p\} T\,\mathrm{d}x \right), \tag{2}$$

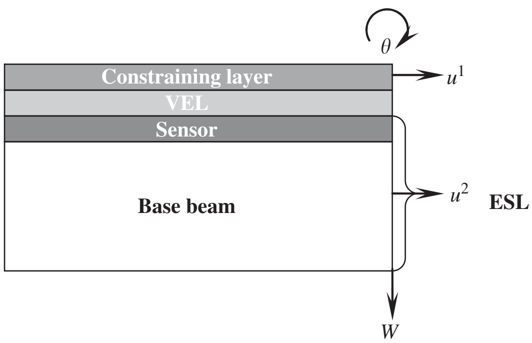

Fig. 1. Beam with hybrid damping treatment.

where $\bar{\phi}_s$, t_s, t_b, b, A_s, ε_{33}, e_{31}, $\{p\}$, T, $[B]$, $\{\delta\}$ are the averaged sensor output voltage over the surface of the sensor, thickness of the sensor, thickness of the base beam, breadth of the beam surface area of the sensor, dielectric constant, stress electric coefficient, pyroelectric coefficient matrix, temperature at any point, matrix of strain displacement, and vector of nodal displacements, respectively. The actuating voltage ϕ_a according to negative feedback velocity algorithm is given by

$$\phi_a = -G\dot{\phi}_s, \tag{3}$$

where ϕ_a, G are the actuating voltage and control gain, respectively. From Eq. (2) we have

$$\phi_a = \frac{Gt_s b}{\varepsilon_{33} A_s}\left(\int_x \left\{ e_{31} \quad \frac{t_b + t_s}{2} e_{31} \quad 0 \quad 0 \quad 0 \right\}[B]\{\dot{\delta}\}\,\mathrm{d}x + \int_x \{p\}\dot{T}\,\mathrm{d}x\right). \tag{4}$$

Since the present analysis is done assuming steady-state temperature field there will be no actuating voltage due to pyroelectric effect. Hence, the actuating voltage is given by

$$\phi_a = \frac{Gt_s b}{\varepsilon_{33} A_s}\int_x \left\{ e_{31} \quad \frac{t_b + t_s}{2} e_{31} \quad 0 \quad 0 \quad 0 \right\}[B]\{\dot{\delta}\}\,\mathrm{d}x, \tag{5}$$

$$\phi_a = \frac{Gt_s b}{\varepsilon_{33} A_s}\int_x \{e_1\}[B]\{\dot{\delta}\}\,\mathrm{d}x = K_{u\phi S}\{\dot{\delta}\}, \tag{6}$$

where $K_{u\phi S} = Gt_s b/\varepsilon_{33} A_s \int_x \{e_1\}[B]\,\mathrm{d}x$ and $\{e_1\} = \{e_{31} \quad ((t_b + t_s)/2)e_{31} \quad 0 \quad 0 \quad 0\}$.

The expression for elemental damping matrix $[C]$ can be obtained from the virtual work expression as [9]

$$[C] = b\int_x [B]^{\mathrm{T}}\{0 \quad 0 \quad 0 \quad 0 \quad e_{31}\}^{\mathrm{T}}\,\mathrm{d}x\, K_{u\phi S}, \tag{7}$$

$$[C] = b\int_x [B]^{\mathrm{T}}\{e_2\}^{\mathrm{T}}\,\mathrm{d}x\, K_{u\phi S} = K_{u\phi A} K_{u\phi S}, \tag{8}$$

where $K_{u\phi A} = b\int_x [B]^{\mathrm{T}}\{e_2\}^{\mathrm{T}}\,\mathrm{d}x$ and $\{e_2\} = \{0 \quad 0 \quad 0 \quad 0 \quad e_{31}\}$.

The matrix of strain displacement $[B]$ is not dependent on temperature. In the present study the influence of initial stress matrix arising out of temperature has been accounted in the vibration study. This gives us the influence of temperature on frequency and damping. For conducting parametric study with respect to temperature, the beam is subjected to a known temperature at one edge and remaining faces are subjected to convection. Two-dimensional heat transfer is considered in the beam. The temperature is assumed to be constant in breadth direction. The boundary condition is clamped–clamped and the dimensions of the beam are similar to those reported in Ref. [6]. From the previous derivation it is evident that piezoelectric actuating force is accounted by considering it as a damping matrix. So, no other externally applied load separately appears in the equation. Effect of applied temperature is felt by considering initial stress matrix, and temperature dependent complex stiffness matrix. The passive and active modal loss factors can be separately obtained by solving two different eigenvalue problems. For obtaining the passive modal loss factor the following eigenvalue problem has to be solved:

$$[[K] + [K_G]]\{\delta_G\} = \varpi^2[M]\{\delta_G\}, \tag{9}$$

where $\varpi^2 = \omega_r^2(1 + \eta_{pr})$ are the complex eigenvalues of the given problem. From this result the natural frequencies ω_r and passive modal loss factor η_{pr} for the rth mode can be obtained. For obtaining the active loss factor the following eigenvalue problem is solved:

$$\begin{bmatrix} [K] + [K_G] & [0] \\ [0] & [M] \end{bmatrix} \begin{Bmatrix} \{\delta_G\} \\ \{\dot{\delta}_G\} \end{Bmatrix} = \lambda \begin{bmatrix} -[C] & -[M] \\ [M] & [0] \end{bmatrix} \begin{Bmatrix} \{\delta_G\} \\ \{\dot{\delta}_G\} \end{Bmatrix}, \tag{10}$$

where the eigenvalues λ are complex conjugate and are given by $\lambda = \lambda_{Rr} \pm \mathrm{i}\lambda_{Ir}$ whose imaginary values will give natural frequencies. The rth modal active loss factor is given by $\eta_{ar} = -\lambda_{Rr}/\sqrt{\lambda_{Rr}^2 + \lambda_{Ir}^2}$. The total loss factor for the rth mode is the sum of active and passive loss factors of the rth mode given by

$$\eta_{Tr} = \eta_{pr} + \eta_{ar}. \tag{11}$$

3. Results and discussion

3.1. Validation

The present computer code is validated for the frequency and damping behavior with the results available in the literature [6]. Hau and Fung [6] have studied the effect of patch configuration on the damping performance of a flexible beam. The frequency values obtained by the current formulation for the configuration given in Ref. [6] tally well with the results of the same as shown in Table 1. Hau and Fung [6] modeled the VEL using GHM method whereas the current work employs complex modulus method. The discrepancy in higher modes can be attributed to the difference in modeling. Due to a difference in procedure adopted for the calculation of damping, the trend in first mode damping values has been validated and the trend is matching well with the results of Ref. [6] as shown in Fig. 2.

3.2. Influence of VEL core thickness

Figs. 3–5 show the variation of loss factor and natural frequency of the beam with respect to temperature. The core material used is EC2216. The first mode passive loss factor increases constantly with temperature. For the higher modes the passive loss factor reaches a maximum and falls down from then onwards mimicking the variation of material loss factor with temperature as given in Ref. [10]. The influence of geometric stiffness matrix is felt on the first mode and hence the damping goes on increasing as the real part of the total stiffness

Table 1
Comparison of natural frequencies

Mode number	Frequency (Hz) from Ref. [6]	Frequency (Hz) present
1	35.50	35.97
2	180.45	177.73
3	480.43	463.19

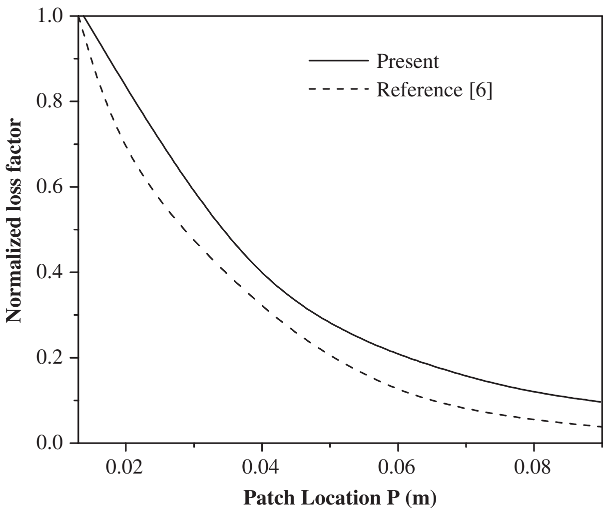

Fig. 2. Variation of first mode loss factor with location of patch P from the present code. For the detail of configuration see Ref. [6].

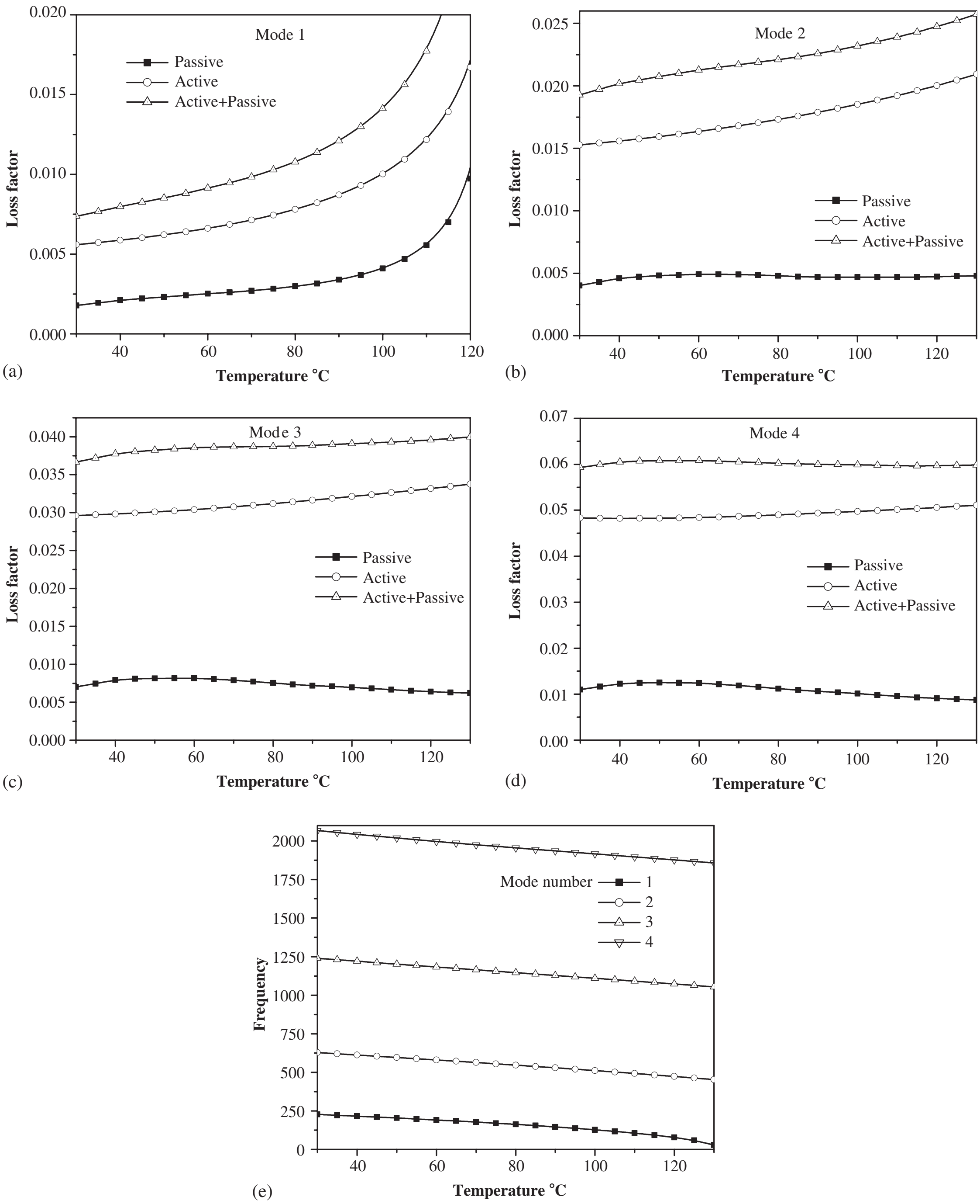

Fig. 3. Variation of loss factor and frequency with tip temperature for a beam with EC2216 as core material. Thickness of the core $= t_c = 0.0025\,\text{m}$.

matrix decreases. In contrast, at higher modes as the buckling temperature is much higher only the material property of the core dominates and hence the damping follows the variation pattern similar to that given in Ref. [10]. As core thickness increases, correspondingly passive damping in the beam increases. As the temperature approaches the first mode buckling temperature the frequency of the first mode tends to zero and loss factor values shoots up. The buckling temperature increases slightly with the increase in core thickness as

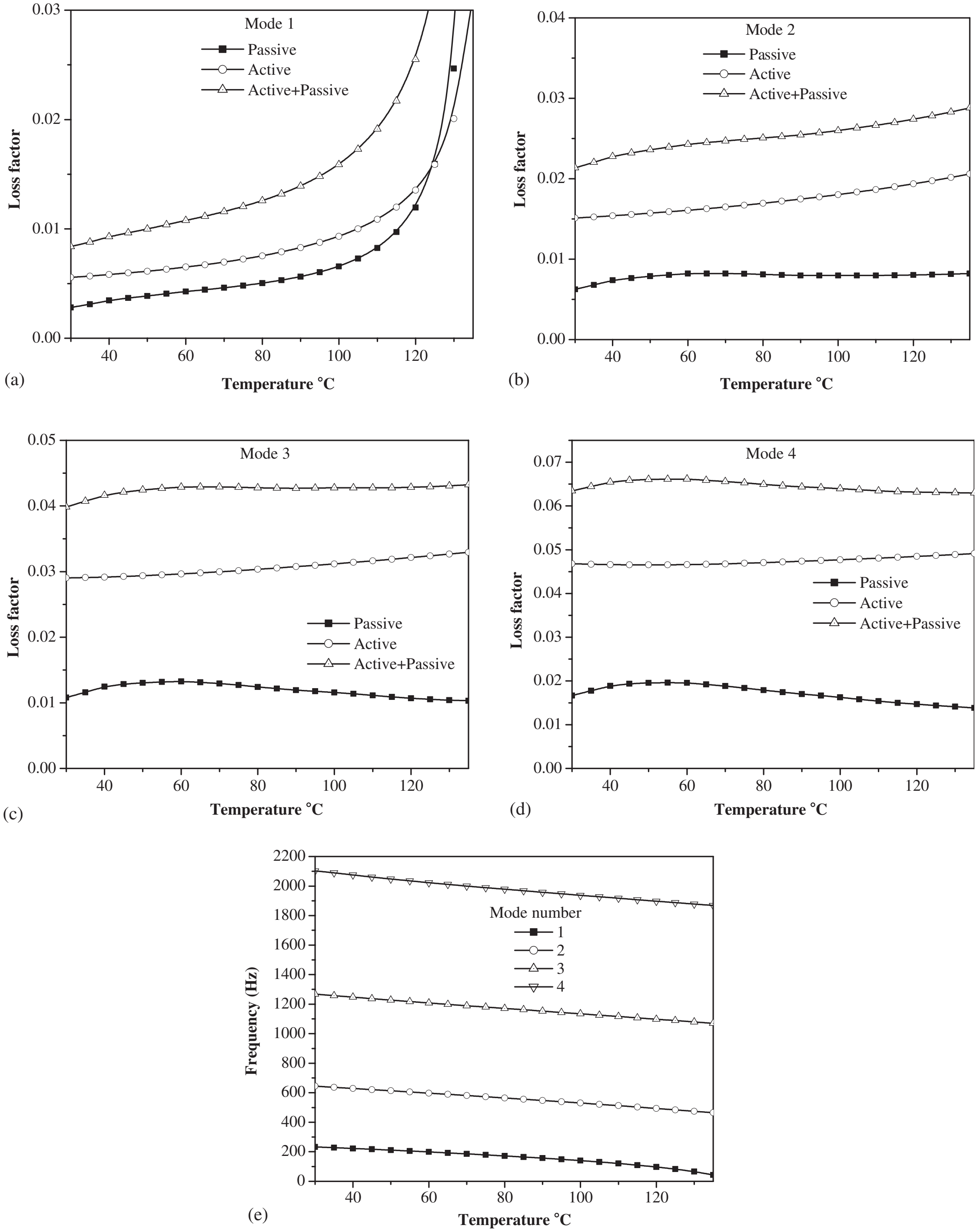

Fig. 4. Variation of loss factor and frequency with tip temperature for a beam with EC2216 as core material. Thickness of the core $= 2t_c = 0.005\,\text{m}$.

expected. The active damping in the system increases with temperature. The contribution of active damping to the total system damping (Active + Passive) increases as the mode number increases. The dominant mechanism of damping in this case is active damping.

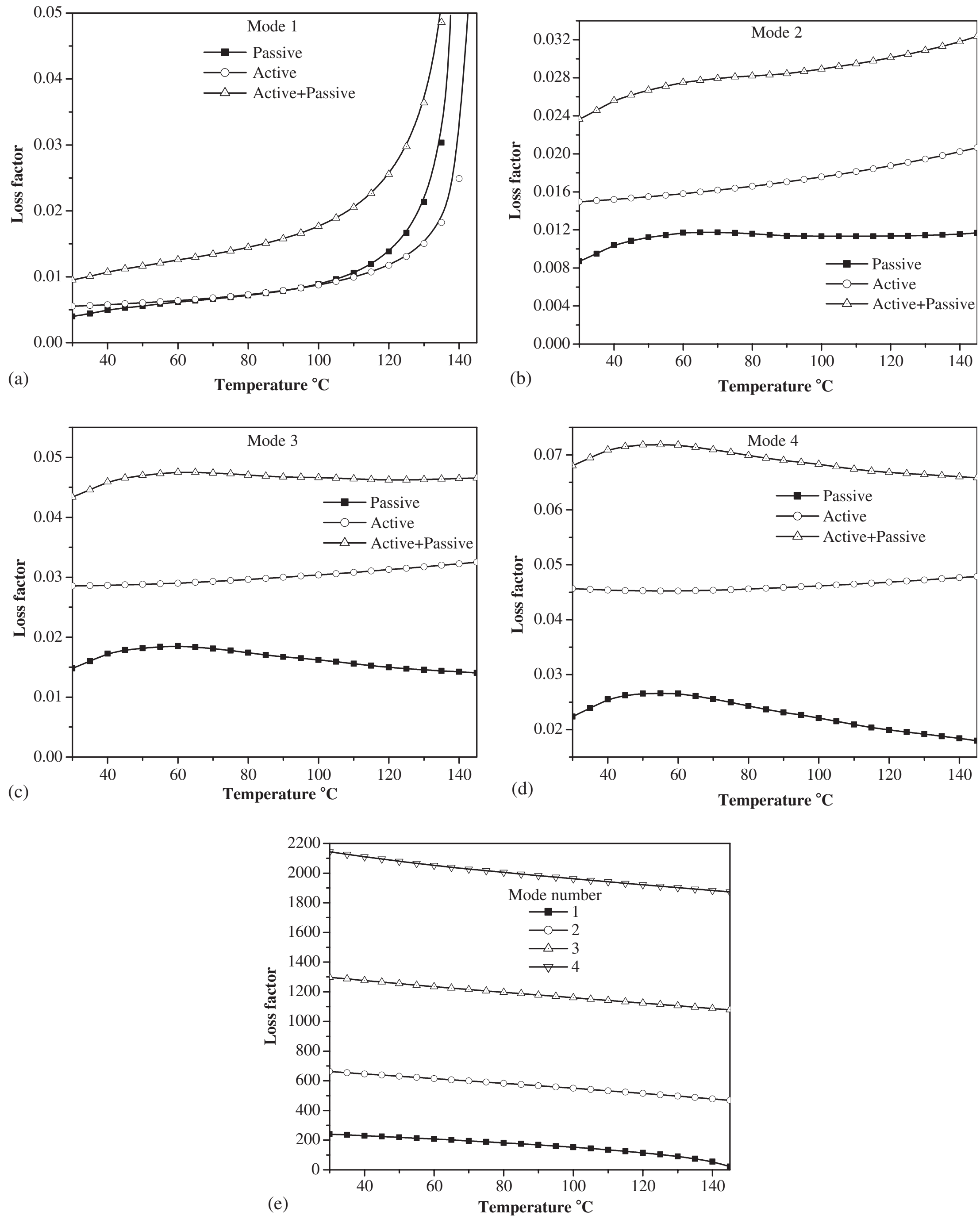

Fig. 5. Variation of loss factor and frequency with tip temperature for a beam with EC2216 as core material. Thickness of the core $= 3t_c = 0.0075\,\mathrm{m}$.

3.3. Influence of VEL core material

Fig. 6 shows the variation of loss factor and natural frequency of the beam with respect to temperature. The core material used is DYAD606. Comparison of Figs. 6 and 3 reveals that core materials plays an important role in the total damping. In general, there is an optimum shear parameter value to have the damping

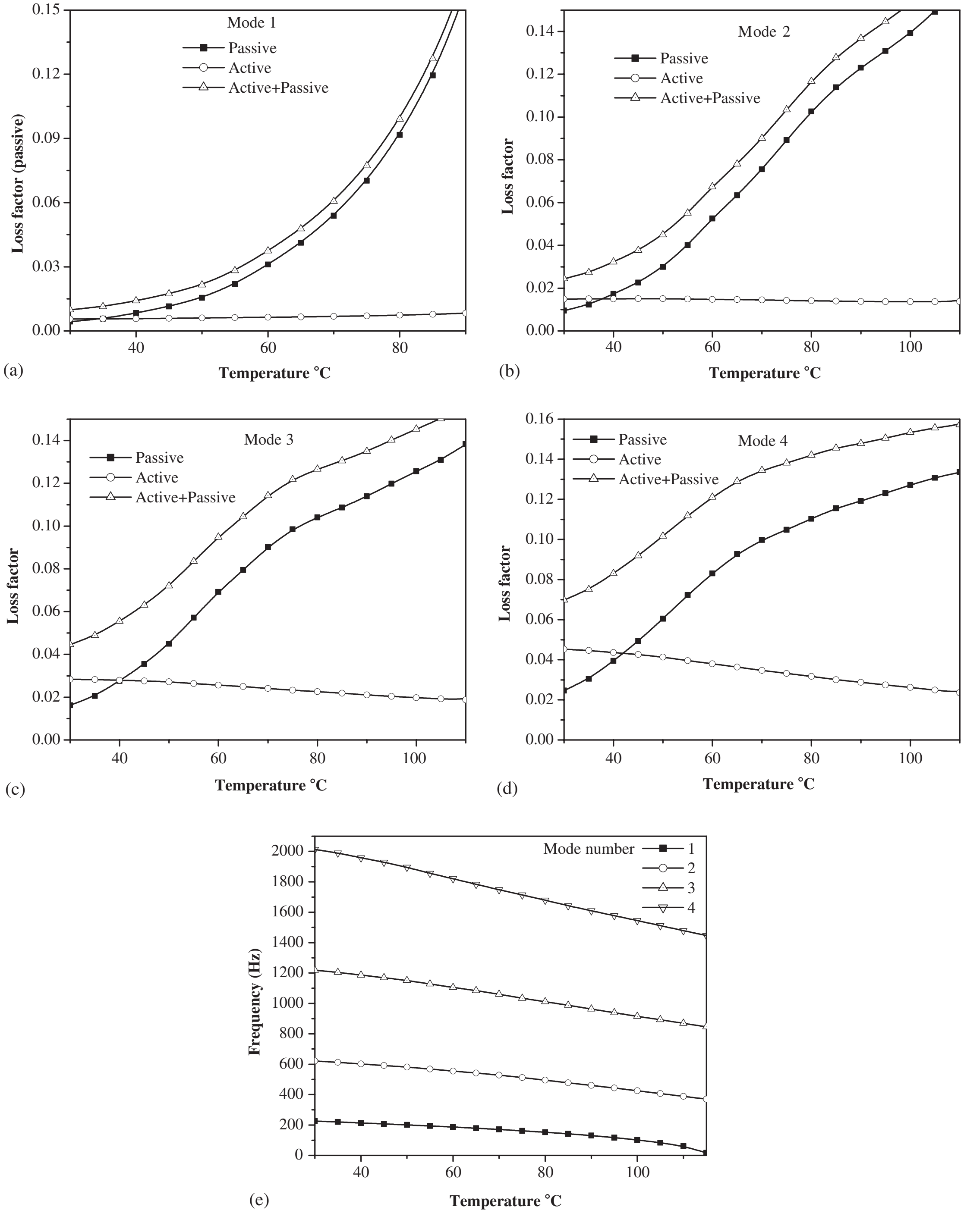

Fig. 6. Variation of loss factor and frequency with tip temperature for a beam with DYAD606 as core material. Thickness of the core $= t_c = 0.0025\,\text{m}$.

maximum. The frequency variation is similar to the case of EC2216. For this beam the passive loss factor for all the modes will increase monotonically with temperature. This behavior can be attributed to the apparently frantic fall of the shear modulus of DYAD606 with temperature. Active damping decreases with temperature. In this case passive damping contributes more to the total damping of the system since DYAD606 has got

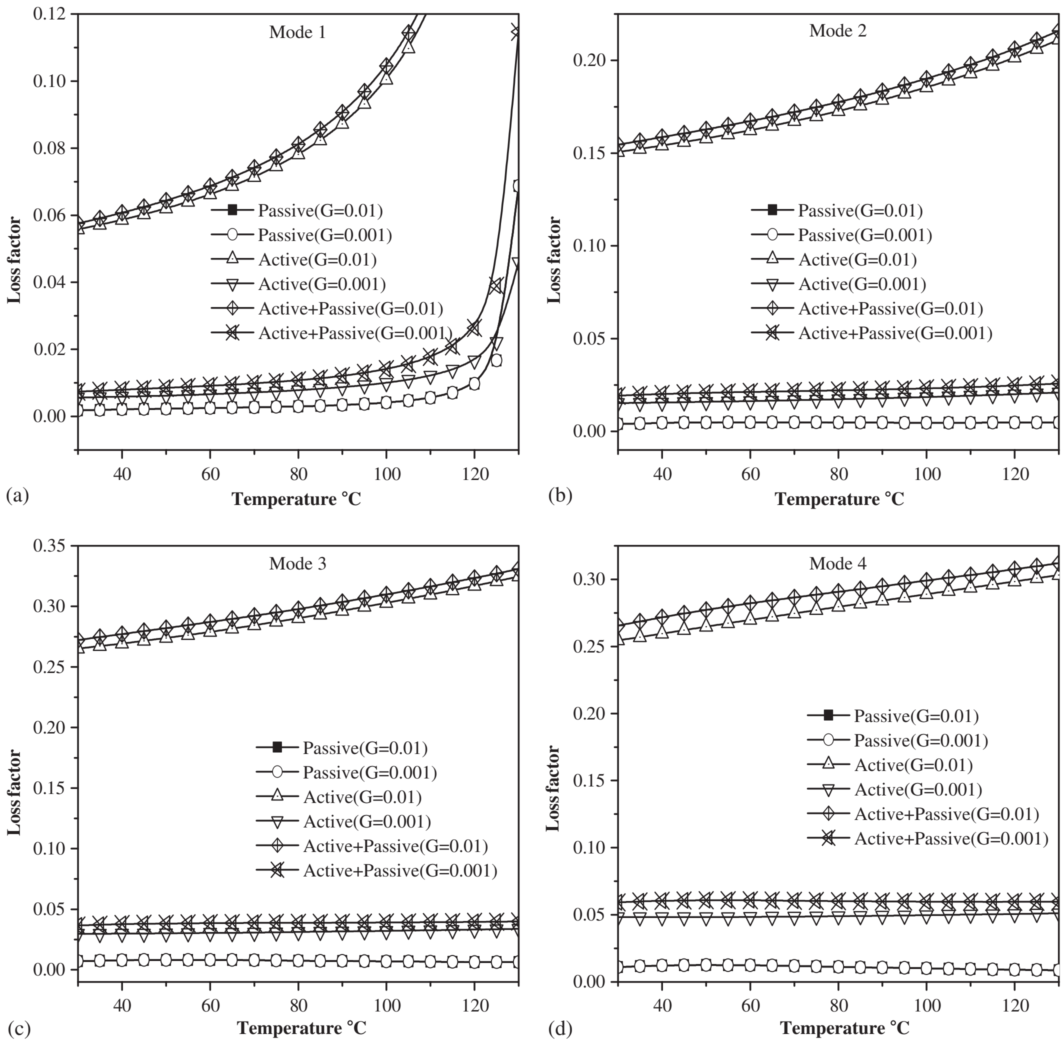

Fig. 7. Comparison of loss factors for two different values of gains.

higher values of material loss modulus compared to EC2216. Slightly lower modulus of DYAD606 compared to EC2216 results in slightly lower buckling temperatures for this beam.

3.4. *Influence of control gain*

Fig. 7 compares the loss factors of the beam for two different control gains. As expected, the increase in control gain results in higher active damping. The active loss factors are almost scaled by the factor by which the control gain increases.

4. Conclusion

In the present study, for the first time in literature active constrained beam subjected to temperature is analyzed for its vibration and damping behavior under thermal environment.

1. In general as core thickness increases passive damping in the beam increases.
2. Near the buckling temperature the loss factor value shoots up.
3. For the cases considered, the active damping is the predominant mechanism of damping.

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